Comparative Analysis of PID, NARMA L-2 and PSO Tuned PID Controllers for Nonlinear Dynamical System

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Abstract: Practically almost all dynamical systems are complex and mathematically ill-defined due to the presence of model nonlinearities and uncertainties. The operation of dynamical systems with suitable adaptive controllers to maintain permissible performance levels under significant unanticipated model nonlinearities and uncertainties are necessary in high performance dynamical systems. In this paper the conventional Proportional Integral Derivative (PID), NARMA L-2, Particle Swarm Optimization tuned PID controllers are analyzed and their effects are compared through performance evaluation of a complex nonlinear dynamic system. The proposed controllers are able to drive uncertain complex dynamical systems with a greater degree of accuracy stably. The performance tests for these controllers were realized in the simulation environment on few selected type of nonlinear dynamical systems. The simulation results validate the superiority of the PSO tuned PID controller over conventional PID, NARMA L-2 controllers in terms of performance abilities and robustness.

Keywords: Proportional-Integral-Derivative Controller, Nonlinear Autoregressive Moving Average, Particle Swarm Optimization, Nonlinear Dynamical System.

1. INTRODUCTION

Most of the physical systems are nonlinear and their analysis is usually complex when compared to the linear systems. Many significant attributes of system performance possibly ignored, if these nonlinear systems are analysed and designed by using linearization techniques. The nonlinear system behaviour is stable for one kind of input and may become unstable for another kind of input. There is no general solution method for nonlinear systems. However, a nonlinear dynamical system can be linearized to some degree, but discontinuity in nonlinear element characteristics such as coulomb friction, dead band, plant saturation, backlash, and hysteresis [1] restrict the linearization of these systems. Hence, development of nonlinear analysis techniques is important to predict the dynamical system performance. Several dynamical systems involve uncertainties in the model parameters due to sluggish or sudden change of plant dynamic parameters. A linear controller based on imprecise values of model parameters may exhibit significance performance degradation or even instability. In many systems, the nonlinearities are deliberately introduced to compensate the model uncertainties into the controller part of the dynamical systems. Moreover, certain other difficulties such as presence of unknown nonlinear function, uncertainty and computational complexity increase the design complexities of the dynamical system. Hence, the control systems are designed with intelligent controllers to deal with the above mentioned complexities and tackled by multi-disciplinary tools. Further
the dynamics of the optimal system parameters are to be changed according to the operating point by the controller. When the time variable is discrete, the nonlinear dynamic system is modelled by difference equations otherwise it is modelled by ordinary differential equations and it is used to design adaptive controllers [2]. The objective of this paper is to compare the performance of nonlinear dynamical system with conventional Proportional–Integral–Derivative (PID) controller to that of the Nonlinear Auto Regressive Moving Average Level-2 (NARMA L-2) and Particle Swarm Optimization (PSO) tuned PID.

For many control problems the conventional PID controllers are accepted by industries due to its simplicity, but still it has limitations like optimality and tuning rules [3-4]. In recent past NARMA L-2 controller were successfully presented to advance the performance of nonlinear dynamical systems, with no overshoot and excellent steady state performance over conventional PID controller [5]. J.Kennedy [6] et.al developed biologically inspired particle swarm optimization algorithm to solve unconstrained single objective function but later on it was modified to unconstrained, single or multi-objective problems. A. Oi [7] et.al suggested the PSO algorithm to search PID parameters for achieving the expected step response by PID tuning tool. P.L.Chen [8] et.al proposed online tuning of PID controller though the set point changes and load disturbance to obtain the good performance and stability. Nema.S and Padhy. P.K [9] presented a novel approach to design of PID controller for two input two output system using particle swarm optimization technique. W.Xin [10] et.al proposed PSO based self-tuning PID controller with variable parameters and reported that the controller is stable and reliable.

This paper is organized in four sections: Section 2 analyzes the modelling of nonlinear dynamical system. Section 3 discusses the three controllers followed by the case study in which each of the controller is applied to the test problem of nonlinear dynamical system. Section 4 simulation results and section 5 is conclusion.

2. MODEL OF NON LINEAR DYNAMICAL SYSTEM

The nonlinear dynamical system (NLDS) is represented using state equations, differential equations or difference equations. The nonlinear dynamical system considered in this paper is represented by the following equation [11], where $y$ is output and $u$ is input of the system:

$$\frac{d^5 y}{dt^5} + 0.7 \frac{dy}{dt} + 0.2y + 0.3 y^3 = u$$  \hspace{1cm} (1)

The state space representation of this system is described using two state variables $x_1$ and $x_2$ such that

$$x_1 = y$$  \hspace{1cm} (2)

$$x_2 = \frac{dy}{dt}$$  \hspace{1cm} (3)

Then,

$$x_1 = \frac{dx_2}{dt} = x_2$$  \hspace{1cm} (4)

$$x_2 = \frac{d^2 y}{dt^2} = -0.7x_2 - 0.2x_1 - 0.3x_1^3 + u$$  \hspace{1cm} (5)

The system can be compactly written as
\[ \dot{x} = f(x) + g(x)u + d \]

Where,

\[ f(x) = \begin{bmatrix} x_2 \\ -0.2x_1 - 0.3x_1^3 - 0.7x_2 \end{bmatrix} \]
\[ g(x) = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \]
\[ d = 0 \]

A Simulink model of the NLDS is shown in figure 1.

![Figure 1. MATLAB Simulink model of the NLDS](image)

The NLDS transient and steady state responses without any controller is shown in figure 2. The actual response is compared with a reference, [12] that represents the actual desired response of the controlled variable. This approach shows the tracking performance of underdamped system and also find the value of the adaptive parameters which are not exactly the same as that of the desired parameters, the tracking is achieved as time progressive tuning of controller.

![Figure 2. NLDS Responses without any controller](image)

**3. DESIGN OF CONTROLLERS FOR NONLINEAR DYNAMICAL SYSTEM**

The controllers for dynamical systems are designed either by conventional or intelligent control techniques. The conventional PID controller is tuned by obeying the heuristic
Zeigler-Nichols rules [13] to attain stable control in the neighborhood of operating point. In conventional automatic control systems, the self-tuning controllers are not yet sufficiently advanced to cater to the effect of parametric changes in long term performance of dynamical systems. In these situations, an intelligent approach for development of dynamical models based on neural network controllers and bio-inspired search algorithms such as adaptive critic design, is currently attracting much renewed interest among the researchers.

3.1. PID Controllers

PID controllers have been used widely for the control of deterministic control systems for which the precise mathematical model can be formulated. The advantage of using a PID controller is that it has a very simple and easy to implement. It is robust and reliable. A conventional PID controller is shown in figure.3 and uses the reference signal to the NLDS, the error between the actual output and the reference signal along with an integral \( K_i \) and differential \( K_d \) coefficient, generate the control signal \( u \) [13] as given by equation (7)

\[
   u = K_p + K_i \int e dt + K_d \frac{de}{dt}
\]

Where \( e(t) = r(t) - y(t) \).

![PID Controller Diagram](image-url)
The conventional PID control is based on linear control theory and hence is effective when used for simple practical linear processes. However when the system under consideration has nonlinearity or the system parameters vary under a wide range, the performance of conventional PID deteriorates considerably. This happens because the reference input given to the system are usually not continuous or smooth as they are subjected to the disturbance or noise signal, while the output of the system is required to be smooth and continuous. Since the smooth continuous output is taken as the direct objective of the output, the inertia influence of the system is neglected which causes unanticipated overshoot oscillations when used practically. In addition, the reference signals are usually un-differentiable signals which makes it problematic to achieve the differential signal of the error. The linear combination of conventional PID causes the conflict between the overshoot variable and the reference. Initially a high value of $K_p$ would intensify the system response however as the error starts reducing, the $K_p$ value must also change automatically so that there is no overshoot \[14\]. Also when the error is decreasing and the rate of change in error is increasing, the $K_p$ must decrease gradually to reduce the overshoot and when the change of error is decreasing $K_p$ must gradually decrease to avoid the overshoot. But due to the linear combination of the PID, this is not possible. Hence we need to find different nonlinear adaptive controllers to control such nonlinear dynamical system. The PID controller is auto tuned by heuristic Zeigler-Nichols rules for continuous time model. The simulation results for the control signal $u(t)$ is shown in figure.4 and the response of NLDS with PID controller is shown in figure.5

![Figure 4 Control signal from PID Controller](image-url)
3.2. NARMA L-2 Controller

Neural networks (NN) have been extensively used in the identification and control of NLDS. Multilayer neural networks have the universal approximation capability which makes it a popular selection for modelling NLDS and for applying general purpose controller for NLDS. The two phases involved in neural network implementation are system identification and control design [15]. In the first phase, an identical NN model of the plant which is desired to be controlled has to be realized. Subsequently, the developed NN model is then used to train the controller. NARMA L-2 controller is simply a readjustment of the plant model that is trained off-line in batch process. It requires the least computation of most neural architectures as the only computation is forward pass through the neural network controller. The only drawback of this form of controller design is that the plant being controlled should be in companion form or must be capable of being approximated by a companion form model [16]. This type of control is also known as feedback linearization control. A standard model to denote general discrete-time nonlinear system is the nonlinear autoregressive-moving average (NARMA) model [16] expressed by equation (8).

\[ y(k + d) = N[y(k), y(k - 1), ..., y(k - n + 1), u(k), u(k - 1), ..., u(k - n + 1)] \tag{8} \]

Where \( u(k) \), \( y(k) \) and \( d \) are the system input, system output and system delay respectively. For the identification phase, to train the NN, the nonlinear function ‘N’ is approximated. If the system output followed reference trajectory \( y(k + d) = y_r(k + d) \), to develop a nonlinear controller [5] by using equation (9)

\[ u(k) = G[y(k), y(k - 1), ..., y(k - n + 1), y_r(k + d), u(k - 1), ..., u(k - m + 1)] \tag{9} \]

Implementation of NARMA L-2 Controller is quite slow due to the dynamic back-propagation training and creation of a function ‘G’ to minimize the mean square error. The solution is to approximate the model by equation (10).

\[ y(k + d) = f[y(k), y(k - 1), ..., y(k - n +1), u(k - 1), ..., u(k - m + 1)] + g[y(k), y(k - 1), ..., y(k - n +1), u(k - 1), ..., u(k - m +1), u(k)] \tag{10} \]

This model is in companion form [16], where the next controller input \( u(k) \) is not confined inside the nonlinearity. The advantage of this form is that the control input that causes the
system output to follow the reference \( y(k + d) = y_r(k + d) \), can be determined. The consequent controller would take the form given in equation (11)

\[
u(k) = \frac{y(k+d) - f[y(k), y(k-1), \ldots, y(k-n+1), u(k-1), \ldots, u(k-n+1)]g[y(k), y(k-1), \ldots, y(k-n+1), u(k-1), \ldots, u(k-n+1)]}{g[y(k), y(k-1), \ldots, y(k-n+1), u(k-1), \ldots, u(k-n+1)]}
\]  

Directly using the above equation the realization of problems [16] is achieved, as the control input \( u(k) \) has to be determined based on the output \( y(k) \), at the same time. Hence, instead of the previous model the revised model is used and given in equation (12).

\[
y(k + d) = f[y(k), y(k-1), \ldots, y(k-n+1), u(k), u(k-1), \ldots, u(k-n+1)]g[y(k), y(k-1), \ldots, y(k-n+1), u(k), \ldots, u(k-n+1)]u(k+1)
\]

Where \( d \geq 2 \). The structure of a NN representation for NARMA L-2 controlled NLDS plant model is shown in the figure.

**3.2.1. NARMA L-2 Controller Design**

Neural networks are widely used now a days for identification of NLDS through NARMA L-2 controller by feedback linearization. Implementation of NARMA L-2 essentially consists of two steps. The first one is the identification of the plant shown in figure 7 which involves developing an approximate NN model of the NLDS.
The second stage is to obtain a NN model of the plant which is used to train the controller. The model of a NARMA L-2 controlled NLDS is shown in Figure 8 and the control signal from NARMA L-2 controller is also shown in figure 9.

System identification mainly consists of gathering sufficient experimental data, estimating the best approximate model from the data and then validating the model with the data. For the plant identification, the controller bounds for the maximum and minimum inputs, outputs as well as the time interval for which the input to the system would remain constant [5,18]. Number of hidden layers required for the neural network is provided and the delays in the output and input of the system are also provided as 4 and 3 respectively. Since
information about the dynamic behaviour of the NLDS is used for system identification, a sufficient number of training samples (10000) were generated with the above specification. The number of epochs within which the mean squared error (MSE) will be minimized was set to 1000; the training algorithm used for the NLDS identification was Levenberg-Marquandt (trainlm) technique. The experimental data used for training the neural network is shown in figure 10. The testing and validation data sets for NARMA L-2 controller are shown in figure 11 and 12, respectively. After the validation of the training and testing data, the best validation performance is 3.0604e-12 as shown in figure 13 and the response of the NARMA L-2 controlled NLDS is shown in figure 14. It is observed from the results that the NLDS system is stable.

Figure 10. Training experimental data set for NARMA L-2 controlled NLDS

Figure 11. Testing data set for NARMA L-2 controlled NLDS
Figure 12. Validation data set for NARMA L-2 controlled NLDS

Figure 13. Best validation performance of NLDS

Figure 14. Response of NARMA L-2 Controlled NLDS
3.3 PSO Tuned PID Controller

Particle swarm optimization (PSO) is a metaheuristic optimization algorithm motivated by swarm intelligence of fish and bird schooling in nature. The multiple agents known as particles, swarm around the search space starting from some initial guess [19, 20]. PSO have some resemblances with genetic algorithms and ant colony algorithm but PSO is much modest because it uses real number randomness and global communication instead of mutation / crossover operators or pheromone. The PSO algorithm is initialized with a group of arbitrary particles in a search space and then the search for optima begins by updating generations. With each iteration the particles in the search space adjust their position by using two best values. The first best comes from the greatest fitness it has achieved so far (self-experience) which is stored as pbest (g*) and the other best is tracked by any particle in the population (social experience) which is called the global best[g] or gbest (xg). When the algorithm finds these two best values, it updates its velocity and position. Let xi and vi be the position vector and the velocity of particle ‘i’ respectively. The innovative velocity vector is determined by the equation (13)

\[ v_{i}^{t+1} = c(v_{i}^{t} + \alpha \epsilon_{1} \odot [g^{*} - x_{i}]) + \beta \epsilon_{2} \odot [x_{g} - x_{i}] \]  

(13)

Here ‘\( \epsilon_{1} \)’ and ‘\( \epsilon_{2} \)’ be two arbitrary vectors taking values between 0 and 1. The Hadamard product [21] of two matrices \( u \odot v \) is defined as entry wise product \( [u \odot v]_{ij} = u_{ij}v_{ij} \). The parameters \( \alpha \) and \( \beta \) are learning parameters heuristically taken as \( \alpha \approx 2 \approx \beta \). The initial locations of all the particles are uniformly distributed such that they can sample over most of the space. The initial velocity \( v_{i}^{t} = \theta \) at the new position can be updated using equation (14) and the \( v_{i}^{t} \) value is bounded in the range of \( [\theta \ v_{max}] \). The flow chart of the proposed PSO tuned PID with NLDS is shown in figure.15.

\[ x_{i}^{t+1} = x_{i}^{t} + v_{i}^{t+1} \]

(14)

The initial values for the PSO algorithm is as follows:

- Population size = 10
- Dimension = 3
- Learning parameter \( \alpha = 2.05 \)
- Learning parameter \( \beta = 2.05 \)
- Constriction factor \( C = 2 \)
- Maximum iterations = 50

The implementation of the PSO algorithm to tune the PID controller can broadcast the current best estimate and gives better and quicker convergence during optimal tracking of the specified desired signal. Here the swarm size, the number of iterations and the learning factors are taken as 20, 50 and 2.05 respectively. The PSO algorithm estimates the best \( K_p \), \( K_i \) and \( K_d \) parameters based on the error signal \( e(t) \). The functional diagram of the PSO tuned PID controller with NLDS shown in figure 16. The PSO uses the NLDS output to search the optimal solution of the parameters \( \Delta K_p \), \( \Delta K_i \) and \( \Delta K_d \) for every discrete step. By knowing these states, the new optimal solution parameters would be added to the earlier PID parameters \( K_p \) (n-1), \( K_i \) (n-1) and \( K_d \) (n-1). The response of the NLDS with optimally tuned PID by PSO is shown in figure 17. The optimally tuned PID using PSO gives the parameters \( K_p = 167.7437 \), \( K_i = 100.68827 \) and \( K_d = 70.7849 \).
Set the range of real controller parameters $K_p, K_i, K_d$

Set the iteration value

Initialize the swarm to form solution space

Evaluate fitness for each particle

Update global and local best

Update velocity and position of each particle

If the maximum value of iterations reached

Store best individual controller parameters

Figure 15. Flow chart of the PSO tuned PID with NLDS

Figure 16. Functional diagram of PSO tuned PID controller with NLDS
4. COMPARISON OF SIMULATION RESULTS

Comparison of the NLDS responses using conventional PID, NARMA L-2 and optimally tuned PID by PSO is carried out through simulation study of NLDS. Since the reference trajectory taken is a model reference for a NLDS, both the error elimination and the transient behaviour of the system has been taken into account. The system is originally unstable due to the presence of disturbances, unmodelled dynamics and parametric variations. Figure 18 shows the system responses with the different types of controllers in respect of the various performance criteria like settling time, peak overshoot, and steady state error.

The simulation results in figure 19 shows the trajectory tracking behaviour of the optimally tuned PID controller by PSO and it is observed from figure 19 that the PSO tuned PID operator of NLDS gives improved response than the either conventional PID and the NARMA L-2 controller for varying reference signal at t=10 sec.
The following performance parameters through the dynamic response analysis of NLDS have been determined by using PID, NARMA L2 and PSO tuned PID controllers and the values are summarized in Table I

<table>
<thead>
<tr>
<th>Controllers</th>
<th>Settling Time (Sec)</th>
<th>Peak Overshoot (%)</th>
<th>Steady state error</th>
</tr>
</thead>
<tbody>
<tr>
<td>PID</td>
<td>3.975</td>
<td>0.0401</td>
<td>0.0002</td>
</tr>
<tr>
<td>NARMA-L2</td>
<td>4.000</td>
<td>-0.0375</td>
<td>0</td>
</tr>
<tr>
<td>PSO tuned PID</td>
<td>3.850</td>
<td>0.0002</td>
<td>0</td>
</tr>
</tbody>
</table>

**5. CONCLUSION**

The conventional PID, NARMA L-2 and optimally tuned PID controller by PSO have been successfully developed and tested for their performance in terms of settling time, peak overshoot and steady state error on a NLDS. Simulation results shows the effectiveness of these controllers for dealing with NLDS for widely varying operating conditions. In comparison with the conventional PID controller, the NARMA L-2 controller has the advantage of a no overshoot, zero steady state error but with the drawback of a higher settling time. On the other hand conventional PID controller tuned using PSO technique with model reference yields better performance as compared to these two controllers.
REFERENCES